Indian J. Phys. **47**, 471–479 (1973)

Effective density of rotating nucleons and moment of inertia calculations of deformed nuclei

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(*Received* 2 *August* 1972)

Following the 'Governer model' of Gupta & Trainor (1969), an expression for the effective density of rotating nucleons is derived. Using experimental values of effective moment of inertia and deformation parameter, effective densities for deformed in nuclei in the regions $152 \leqslant A \leqslant 190$ and $A \geqslant 222$ are calculated and are found to be 0 95 and 1·2, respectively On the basis of the same model, a simple expression has been derived for the effective moment of inertia of even-even deformed nuclei. The derived expression gives satisfactory agreement with experimental values.

INTRODUCTION

Hofstador (1956) has shown that the charge density of nucleus is well represented by a function which is almost constant inside the nucleus but falls to zero within a region 2–3 fm. This is confirmed by experiments on μ-mesonic atoms and optical model analysis. Mass distribution has been determined from nucleon scattering experiments using optical potentials (Holmquist & Weidling 1968, Greenlees *et al* 1968). Recently Nilsson (1970) has calculated parameters of proton and neutron distributions in heavy deformed nuclei using a deformed single particle model.

The density distribution at nuclear surface can be used to study the applicability of model effective interactions for nuclear structure calculations. The Hartree-Fock calculations in heavy nuclei are rather complicated and hence local density approximation is commonly used for these studies. Such calculations have been reported by Nemeth & Bethe (1968) for semi-infinite nuclear matter and by Bruckner *et al* (1968) and also by Donnelly (1968, 1969) for finite nuclei. Greenlees *et al* (1968) have studied the neutron density distribution in medium and heavy nuclei with a reformulated optical model. More recently the distribution of proton and neutron densities in some nuclei by variational methods has also been studied. Thomas-Fermi theory has also been used (Dahll & Warke 1970) to calculate density distributions of neutrons and protons in some finite nuclei.

In case of rotating nuclei since the rigid body moment of inertia is about twice the ground state value, it is clear that not all the nucleons take part in rotation Gupta & Trainor (1968, 1969, 1970) and Gupta (1967, 1969a) have considered the portion of the nucleus taking part in rotation as the mass lying outside a rotationally invariant core (RIC), whereas, Krutov (1968) has defined the collective rotational motion as charge of mass density distribution in time such that any motion, which does not change the mass density distribution, does not also contribute to the energy of collective motion

In this study we have calculated, using classical methods, the effective densities of the rotating mass of the nuklei in the regions $152 \leqslant A \leqslant 190$ and $A \geqslant 222$ from experimental value of effective or ground state moment of inertia. Again an expression has been derived for the effective moment of inertia using the assumption of the rotationally invariant core of the 'Governer model' (Gupta & Trainor 1968, 1969, 1970)

Formulation

(a) According to this model the rotating nucleus consists of the RIC and another part which lies outside the RIC and takes part in rotation Semi-major axis, semi-minor axis and equivalent nuclear radius are denoted by a, b and R, respectively. The nucleus is assumed to possess prolate deformation and volume conservation. Since the number of nucleons taking part in rotation is only a fraction of the mass number A of nucleus and since the rotating mass and the stationary mass (which lies in the RIC) occupy different volumes, the densities of the rotating and stationary parts must also be different and can be calculated as follows.

Volume of the rotating mass

$$V = \frac{4\pi}{3}(a-b)b^2$$

and volume of the RIC

$$V_s = \frac{4\pi}{3}b^3.$$

Thus

$$\frac{V}{V_s} = \frac{a-b}{b} = \frac{\beta}{1{\cdot}06-0{\cdot}33\beta+0{\cdot}1\beta^2} \qquad \dots (1)$$

using

$$a/R = 1+\frac{2\eta}{3}+\frac{\eta^2}{9}$$

and

$$b/R = 1-\frac{\eta}{3}+\frac{\eta^2}{9}$$

given by Brancazio & Cameron (1969); where

$$\eta = \frac{a-b}{R},$$

and β is deformation parameter. Moment of inertia of a rigid sphere of radius R and mass A_m is

$$I_r = \frac{2}{5} A_m R^2$$

which gives

$$I_r = \frac{2}{5} A_m \frac{a^2}{1+1{\cdot}26\beta+0{\cdot}61\beta^2} \quad \ldots \quad (2)$$

From Non-rigid-rotator model of Gupta (1967, 1969a), $I_0 = mr_0^2$ where m = one quarter mass (which takes part in rotation) and r_0 = the distance between the two centres of mass of the two segments of a nucleus divided into two by a plane through the axis of rotation and perpendicular to the symmetry axis. Using above equation

$$\frac{A_m-m}{m} = \frac{1{\cdot}4(1+1{\cdot}26\beta+0{\cdot}61\beta^2)}{I_0/I_r}-1 \quad \ldots \quad (3)$$

Evidently A_m-m is the mass lying in the RIC and occupying volume V_s whereas rotating mass m has a volume V. Thus

$$\frac{d_s}{d} = \frac{A_m-m}{m} \times \frac{V}{V_s}$$

and

$$\frac{d}{d_s} = \frac{I_0}{I_r} \frac{(1{\cdot}06-0{\cdot}33\beta+0{\cdot}1\beta^2)}{\beta[1{\cdot}4(1+1{\cdot}26\beta+0{\cdot}61\beta^2)-I_0/I_r]} \quad \ldots \quad (4)$$

where d_s and d are the densities of masses in volumes V_s and V, respectively.

(b) From

$$V = \frac{4\pi}{3}(a-b)b^2,$$

the mass taking part in rotation

$$m = \frac{4\pi}{3}(a-b)b^2.d.$$

Thus

$$I_0 = mr_0^2 = \frac{3}{4}\pi d(a-b)a^2b^2 \quad \ldots \quad (5)$$

Using expressions of Brancazio & Cameron (1969),

$$I_0 = \frac{3}{4}\,\frac{\pi\beta d}{1{\cdot}06}\,R^5(1+1{\cdot}26\beta+0{\cdot}597\beta^2)(1-0{\cdot}56\beta+0{\cdot}278\beta^2)$$

and

$$I_0/I_r = 1{\cdot}32\beta(1+0{\cdot}39\beta+0{\cdot}05\beta^2) \qquad \ldots \quad (6)$$

assuming constant density distribution.

Results and Discussion

From relation (3) using experimental values of I_0/I_r (Davidson 1965), the mass m of nucleons taking part in rotation is determined Taking the mass of the nucleus equal to its mass number, the number of nucleons taking part in rotation is found out Again from equation (4), d/d_s is calculated This ratio can be taken as one representing the effective density of rotating mass Table 1 gives these ratios for even-even deformed nuclei in the regions $152 \leqslant A \leqslant 190$ and $A \geqslant 222$

Figure 1 shows the variation of d/d_s with $1/\beta$, whereas, figure 2 shows the number of nucleons N against deformation parameter β Using relation (6) the values of I_0/I_r have been calculated Experimental values of β have also been taken from Davidson (1965) Figure 3 shows the varitation of ratio of experimental and calculated values of I_0/I_r with β and can be used to test the validity of proposed relation (6).

In a spherical nucleus the nuclear matter is of roughly uniform density throughout the nuclear volume but, as is clear from table 1, in rotating nuclei the density in the rotating part is different from that in the RJC. Thus in the rare-earth region it lies between 0·80 and 0·95, whereas, in the actinide region it varies between 1·1 and 1·3.

From figure 1 it is clear that best results, separately for the two regions, can be obtained by plotting two different curves, one in each region It should be noted that the curves show only a qualitative relation between effective density and deformation parameter and the quantitative relation is given by relation (4). Since I_0/I_r is also a variable factor and functions of β occur in it, the curves in figure 1, give only average values of d/d_s pertinent to each region.

The shapes of the two curves in figure 1 can be explained on the basis of the relation

$$\frac{V}{V_s} = \frac{\beta}{1{\cdot}06-0{\cdot}33\beta+0{\cdot}1\beta^2}.$$

Whereas, in the actinide region I_0/I_r (a measure of the mass participating in rotation) are almost equal to that in the rare-earth region, the deformation parameter β in the former region are, on the average, 25% less than that in the latter.

TABLE 1

Nucleus	(Expt.)	I_0/I_r (Expt.)	d/d_s	
Rare-earth Region				
Sm^{152}	.290	.380	.73	28
Sm^{154}	.336	.551	1.02	40
Gd^{154}	.280	.373	.80	34
Gd^{156}	.320	.498	.97	38
Gd^{158}	.346	.547	.78	41
Dy^{160}	.301	.490		39
Dy^{162}	.320	.512	.96	39
Dy^{164}	.334	.558	1.02	44
Er^{164}	.306	.456	.96	37
Er^{166}	.323	.496	.94	40
Er^{168}	.320	496	.96	40
Er^{170}	.310	.484	.97	41
Yb^{170}	.304	.455		37
Yb^{172}	.311	.477		37
Yb^{174}	.308	.475	.97	40
Yb^{176}	.301	.445	.89	35
Hf^{176}	.300	.410	.82	35
Hf^{178}	.310	.380	.72	33
Hf^{180}	.370	.380	.88	36
W^{182}	.280	.340	.73	31
W184	.250	.310	.88	30
W^{186}	.259	.272	.63	26
Os^{186}	.201	.247	.80	31
Os^{188}	.191	.214	.72	23
Os^{190}	.180	.180	.64	19
Actinide Region				
Ra^{222}	.184	.223	.80	18
Ra^{224}	.171	.291	1 18	37
Ra^{226}	.197	.351	1.45	45
Ra^{228}	.212	.400	1.32	50
Th^{226}	.220	.330	1 00	37
Th^{228}	.225	.403	1.24	50
Th^{230}	.233	.433	1.31	53
Th^{232}	.243	.450	1 27	56
Th^{234}	.233	.467	1 41	57
U^{230}	.245	.443	1 26	56
U^{232}	.257	.470	1.24	56
U^{234}	.251	.516	1.29	58
U^{236}	.263	.485	1.26	59
U^{238}	.268	.480	1.18	61
Pu^{238}	.271	.493	1 18	62
Pu^{240}	.278	.488	1.06	59

Thus the volume available for the rotating mass in the actinide region, being about 25% less than that for rare-earth region, the density in the former region is larger by about the same amount. The rapid increase of d/d_s for the lower curve, in the range $1/\beta = 0{\cdot}325$ to $1/\beta = 0{\cdot}375$ can be attributed to the rapid increase of I_0/I_r with β in this region as evident from table 1. After reaching a constant deformation value of about 0·33, the lower curve tends to be parallel to the $1/\beta$-axis, denoting the maximum effective density in the rare-earth region of nearly 0·95 Similarly, the minimum effective density is about 0·65 and this value has been used (Gupta 1969b) as the normalized constant density to account for the fact that both rectangular (Krutov 1968) and trapezoidal density (Krutov 1969) distribution give practically identical results for effective moment of mertia.

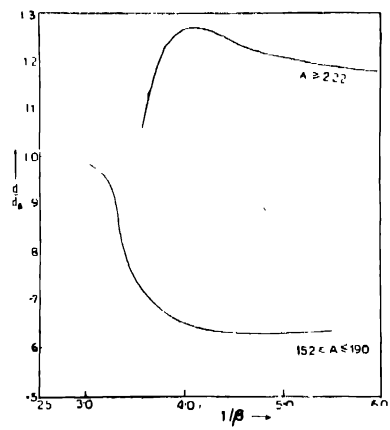

Figure 1 Variation of calculated values of d/d_s with the factor $1/\beta$. Legend to the two curves as shown in the figure.

That the curve in figure 1 for the region $A \geqslant 222$ first reaches a maximum and then shows a decrease in d/d_s with increase in β may be explained by the fact that whereas β for nuclei in this region goes on increasing (from Ra^{222} to Pu^{240}) the I_0/I_r reaches a maximum at U^{234} and then decreases This behaviour should be compared with the observed linear variation of I_0/I_r with β, for β between 0·27 and 0·31 in the rare-earth region, to understand the behaviour of the two curves.

In calculating the number of nucleons participating in rotation from equation (4), the mass of the deformed nucleus has been replaced by A, the mass number, and the value of m obtained has been replaced by the nearest integer.

This has been considered a reasonable approximation From figure 2 it is clear that a linear relationship between N and β in each region of deformation exists. However, it has been found impracticable to draw a single curve through the points representing in both regions; hence two separate curves (in conformity with the trend observed in figure 1). It is evident from table 1 that the number of nucleons taking part in relation is between 1/4th and 1/5th of the mass number A, in both the regions. The linear relationship between N and β can be explained by assigning average effective densities 0·95 and 1·25 to the rare-earth and actinide regions respectively. As β increases, the volume available for rotating mass increases, thereby, increasing N

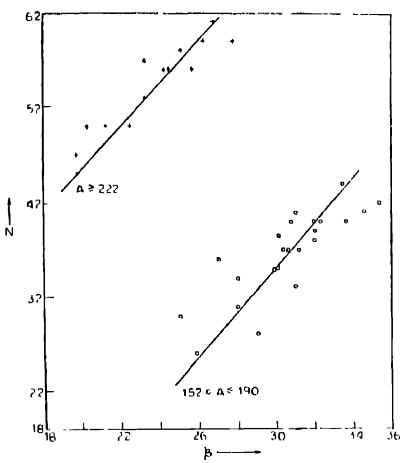

Figure 2. Plot of the calculated number of rotating nucleons N with deformation parameter β. Legend to the two curves as shown in the figure.

It has been observed (Gupta 1969b) that if the radius of RIC is taken as equal to the semi-minor axis of the nucleus then for ground state the number of nucleons in RIC correspond approximately to the major closed shell configuration for the beginning of the rare-earth region, and to this number plus the number in closed subshells through the remainder of the region. This result is independent of assumption regarding density distribution. However, here even odd numbers of nucleons taking part in rotation have been observed This seems to indicate that RIC may contain odd numbers of nucleons also.

It is seen that for nuclei just outside regions $152 \leqslant A \leqslant 190$ and $A \geqslant 222$, the effective density and the number N are unusually small. This arises from the fact that, whereas, for typical deformed nuclei the ratios I_0/I_r are usually 0·5 and 0·4, respectively in the two regions, they are only about 0·2 for these nuclei For this reason, these nuclei have been excluded from calculations though their deformation parameters have values as high as 0·18.

It has been observed that using 1·25 as the average effective density for the region $A \geqslant 222$ in calculations for effective moment of inertia, yields values in excellent agreement with experiment Similarly, it is expected that using constant density (which is very nearly equal to the average effective density 0·95, calculated in this work for nuclei with $152 \leqslant A \leqslant 190$), distribution in calculations for effective moment of inertia for rare-earth nuclei will also give values very near to experimental ones

From figure 3, it is evident that the overall agreement between theory and experiment is good. Whereas, in the actinide region deviations of the order of 15–20% occur, the agreement is excellent in the rare-earth elements It is quite interesting to observe, that using $2/5A\ R^2$ instead of $2/5A\ R^2(1+0{\cdot}31\beta)$ for the regid body moment of inertia introduces an error between 6–10%, depending upon the magnitude of deformation Thus, the results of Gupta (1969b) are perhaps overestimated by this amount.

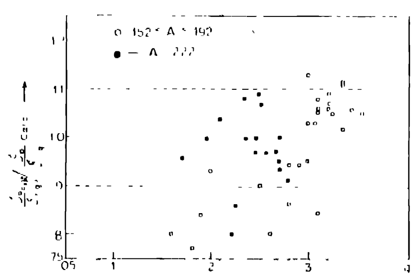

Figure 3. Variation of the ratio of experimental and calculated values of I_0/I with β. Legend to different regions of deformation as shown in the figure.

Whereas, retaining the constant density distribution for the rare-earth region is justified by satisfactory agreement between theory and experiment. the experimental points in the actinide region can be reproduced faithfully if instead of constant density distribution, an effective density of about 1·25 is used for this region as discussed earlier.

ACKNOWLEDGEMENTS

The author is grateful to Dr. O N. Kaul for encouragement and guidance and to Principal R. A. Deshpande of Government Engineering College, Bilaspur, M.P. for providing necessary facilities. The help of B. S Rai in calculations is gratefully acknowledged. Finally the author wishes to thank the Council of Scientific & Industrial Research for financial assistance during author's stay at Government Engineering College, Bilaspur.

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